Exercise 20

Compute the left and right Riemann sums—L_4 and R_4, respectively—for $f(x) = (2 - |x|)$ on $[-2, 2]$. Compute their average value and compare it with the area under the graph of f.

Solution

For $n = 4$, the left Riemann sum is taken from 0 to 3.

$$\begin{aligned}
\int_{-2}^{2} (2 - |x|)\, dx \approx \sum_{i=0}^{3} (2 - |x_i|)\Delta x &= \sum_{i=0}^{3} (2 - |-2 + i\Delta x|)\Delta x \\
&= \sum_{i=0}^{3} \left\{ 2 - \left| -2 + i \left[\frac{2 - (-2)}{4} \right] \right| \right\} \left[\frac{2 - (-2)}{4} \right] \\
&= \sum_{i=0}^{3} [2 - |-2 + i(1)|](1) \\
&= \sum_{i=0}^{3} (2 - |-2 + i|) \\
&= (2 - |-2 + 0|) + (2 - |-2 + 1|) + (2 - |-2 + 2|) + (2 - |-2 + 3|) \\
&= 8 - |-2| - |-1| - |0| - |1| \\
&= 8 - 2 - 1 - 0 - 1 \\
&= 4
\end{aligned}$$

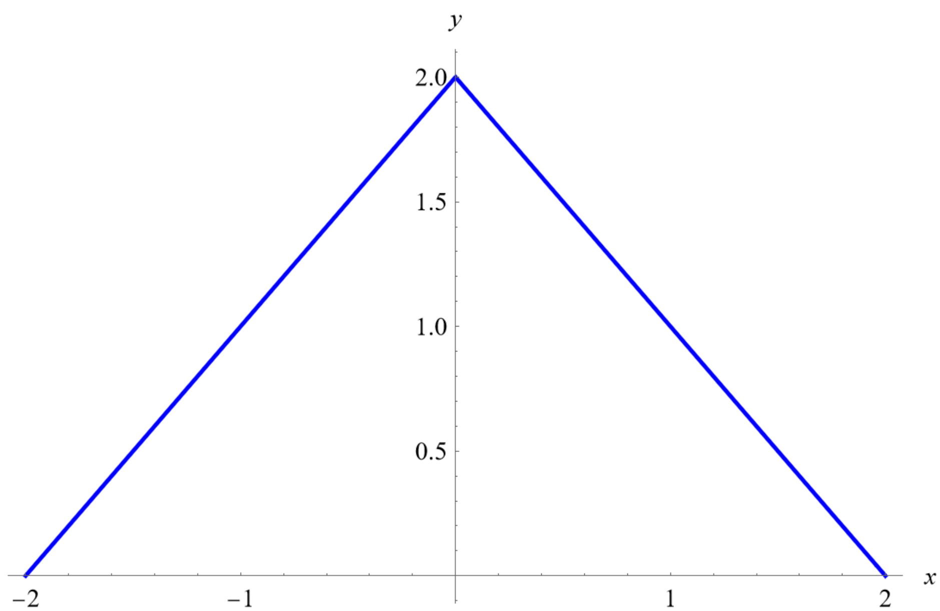
For $n = 4$, the right Riemann sum is taken from 1 to 4.

$$\begin{aligned}
\int_{-2}^{2} (2 - |x|)\, dx \approx \sum_{i=1}^{4} (2 - |x_i|)\Delta x &= \sum_{i=1}^{4} (2 - |-2 + i\Delta x|)\Delta x \\
&= \sum_{i=1}^{4} \left\{ 2 - \left| -2 + i \left[\frac{2 - (-2)}{4} \right] \right| \right\} \left[\frac{2 - (-2)}{4} \right] \\
&= \sum_{i=1}^{4} [2 - |-2 + i(1)|](1) \\
&= \sum_{i=1}^{4} (2 - |-2 + i|) \\
&= (2 - |-2 + 1|) + (2 - |-2 + 2|) + (2 - |-2 + 3|) + (2 - |-2 + 4|) \\
&= 8 - |-1| - |0| - |1| - |2| \\
&= 8 - 1 - 0 - 1 - 2 \\
&= 4
\end{aligned}$$

The average of the two Riemann sums is

$$\frac{2+2}{2} = 2.$$

Now plot the graph of $f(x) = 2 - |x|$ versus x.



The area under the graph is the sum of two triangles with base 2 and height 2.

$$A = \frac{1}{2}(2)(2) + \frac{1}{2}(2)(2) = 2 + 2 = 4$$